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ABSTRACT

Over the past few years a substantial body of research has yielded evidence that the semantic structure of elementary arithmetic word problems influences children's strategies to solve them. The study reported in this document considered three other task characteristics in addition to semantic structure: (1) the order of presentation of the two given numbers; (2) the order of presentation of the two given sets; and (3) the size of the difference between the two given numbers. A paper-and-pencil test on word problems involving addition and subtraction was administered to 85 children in four second-grade classes. Twenty pupils in first-grade classes were individually interviewed. Evidence was found in favor of the impact on children's strategy choice of the first and second variables; however, the data do not support the importance of the third factor. (Three pages of references and 12 tables outlining results are provided). (MNS)

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THE INFLUENCE OF SOME NON-SEMANTIC FACTORS ON SOLVING ADDITION AND SUBTRACTION WORD PROBLEMS

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THE INFLUENCE OF SOME NON-SEMANTIC FACTORS ON SOLVING ADDITION AND SUBTRACTION WORD PROBLEMS

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1. Introduction

Over the past few years a considerable number of studies has been undertaken concerning young children's problem solving on simple addition and subtraction word problems. Two main categories of investigations can be distinguished. Some researchers have collected empirical data concerning the level of difficulty of different types of word problems, children's solution strategies, and the nature of their errors (see e.g. Carpenter & Moser, 1982, 1984; Lindvall & Tamburino, 1981; Morales, Shute & Pellegrino, 1985). Other investigators were involved in building computer-implemented models constituting a well-founded account of the internal processes and cognitive structures underlying children's performances on verbal problems (Briars & Larkin, 1984; Dellarosa, 1986; Kintsch & Greeno, 1985; Riley, Greeno & Heller, 1983). Until now this strand of research has been mainly focused at the influence of one particular kind of task characteristics on children's problem solving, namely the semantic relations among the quantities described in the problem (Change, Combine and Compare) together with the identity of the unknown quantity.

The results of research in our Center over the past few years are generally consistent with the findings of these American investigations concerning the effects of those task characteristics on the relative difficulty of elementary addition and subtraction word problems, on the strategies children use to solve them, and on the nature of their errors (for an overview see De Corte & Verschaffel, 1987). However, our work also suggests that - in addition to the semantic structure - some other task characteristics may also have an important

effect on children's problem-solving processes. In the present paper the results of a recent study are presented in which - together with the semantic structure - three other task characteristics were involved, namely the order of presentation of the two given numbers, the size of the difference between those numbers, and the order of presentation of the two given sets. The study yielded systematic data on the influence of these factors on the same three aspects of performance, namely problem difficulty, typical errors and strategies. However, in the present paper we focus on their effects on children's solution strategies.

2. Children's solution strategies on simple addition and subtraction problems

Since several years we are working on a research project in which an attempt is being made to acquire a better understanding of the development of young children's problem solving skills and processes (De Corte & Verschaffel, 1987). In one longitudinal investigation thirty first graders were individually interviewed three times during the school year: each time they were administered eight elementary arithmetic word problems: four addition (Change 1, Change 6, Combine 1 and Compare 3) and four subtraction problems (Change 2, Change 3, Combine 2 and Compare 1).

Children's solution strategies for elementary addition and subtraction word problems were first categorized into three main categories representing three levels of internalisation: (1) material strategies based on direct modeling of the quantities in the problem with fingers or physical objects; (2) verbal strategies based on the use of counting sequences, and (3) mental strategies based on recalled number facts (Carpenter & Moser, 1982, 1984; De Corte & Verschaffel, in press). Within each of these three levels of internalisation, further distinctions were made for addition and subtraction problems respectively.

With regard to addition problems, we distinguished between strategies in which the child begins with the first of the two given numbers (F-strategies) and strategies starting with the second one (S-strategies). Examples of F- and

S-strategies at the material, the verbal and the mental level are given in Table 1 (1).

Table 1

For problems in which the smaller number is given first, S-strategies are more efficient, especially at the verbal level. By disregarding the given order of the addends and starting with the larger one, the number of steps in the cognitively demanding "double count" is reduced to a minimum (Baroody & Gannon, 1984; Carpenter & Moser, 1982).

With respect to subtraction problems, there are direct subtractive and indirect additive solution strategies. In a direct subtractive (DS) strategy the answer is found by subtracting the smaller given number from the larger one; in an indirect additive (IA) strategy, the child determines what quantity the smaller given number must be added with to obtain the larger one. Table 2 shows examples of DS- and IA-strategies at each of the three levels of internalisation.

Table 2

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3. Effects of semantic problem structure on children's solution strategies

We now briefly mention some main results of that study. First, as one would expect, we found an obvious development in the level of internalisation of children's solution strategies: during the first interview they solved the word problems mainly using material and verbal counting strategies; subsequently they shifted to mental solution strategies based on known number facts.

Second, children's solution strategies for elementary arithmetic word problems were significantly influenced by the semantic structure underlying these problems. More specifically, they tended to solve each problem with that kind of strategy that most closely models its semantic structure. Let us

describe exemplarily some specific findings for addition and subtraction problems respectively (for more details we refer to De Corte & Verschaffel, in press).

We found an obvious relationship between the semantic structure of addition word problems and the kind of verbal counting strategies children used to solve then. In Table 1 verbal counting strategies starting from the first addend (F-strategies) and from the second one (S-strategies) are distinguished. Interestingly, the Combine 1 problem ("Pete has 3 apples; Ann has 7 apples; how many apples do they have altogether ?") elicited much more S-strategies than F-strategies; for the Change 1 problem ("Pete had 3 apples; Ann gave him 5 more apples; how many apples does Pete have now ?") we found the opposite.

The analysis of children's mental strategies on addition problems revealed the same relationship between semantic structure and solution procedure for children operating at this highest level of internalisation. Indeed, the majority of the mental problem solvers applied an S-strategy to solve the Combine 1 problem; the Change 1 problem elicited again much more F- than S-strategies.

We have the following hypothetical explanation for this finding. To obtain a mapping between the mental representation of an addition problem and a verbal or mental S-strategy, the child must interchange the two given quantities in his problem representation. This reorganisation of the original problem representation is less likely to occur when the function of the two given quantities in the problem differs as in Change 1 problems (start set and change set respectively) than when they have the same function as in Combine 1 problems (where both are subsets) (see also Fuson, 1979).

Significant relationships between the semantic structure and children's solution strategies were also found for subtraction problems. We illustrate this with respect to the Change 2 ("Pete had 6 apples; he gave 2 apples to Ann; how many apples does Pete have left ?") and the Change 3 problem ("Pete had 3 apples; Ann gave him some more apples; now Pete has 10 apples; how many apples did Ann gave to Pete ?") in our study.

At the material level, almost all children used the direct subtractive strategy separating from to solve the Change 2 problem: they constructed a set

of six blocks, then took away two blocks, and finally counted the remaining blocks. For the Change 3 problem, the indirect additive strategy adding on was the most frequent material solution strategy: the children constructed a set of three blocks, then added blocks until the set contained ten blocks, and finally counted the number of blocks added.

This tendency to use a strategy modeling the problem structure not only occurred at the material stage, but continued as children shifted to more abstract verbal counting strategies. However, the effect was less straightforward than at the lowest level of internalisation. Indeed, more children used the counting-up-from-given strategy (the verbal counting parallel of the material adding-on strategy) than the counting-down-from-given strategy (the verbal counting parallel of the material separating-from strategy) for Change 2 problems. The surprisingly infrequent use of the counting-down-from-given strategy may be due to young children's tendency to avoid the particular difficult procedure of counting backwards a given number of steps (Carpenter & Moser, 1982, 1984).

Finally, we were able - in contrast to Carpenter and Moser (1982, 1984) - to show that the semantic structure of subtraction problems also influences the strategy choice of children operating at the mental level. For example, during the third interview the Change 2 problem ("Pete had 12 apples; he gave 4 apples to Ann; how many apples does Pete have now ?") was solved much more frequently with a direct subtractive mental strategy (e.g. "12 minus 2 minus 2 is 8") than with an indirect additive strategy ("4 plus 6 plus 2 equals 12, so the answer is 8"). In contrast, the Change 3 problem ("Pete had 5 apples; Ann gave Pete some more apples; now Pete has 14 apples; how many apples did Ann give to him ?") was never solved with a direct subtractive strategy; all mental problem solvers applied an indirect additive strategy.

4. In search of non-semantic factors affecting solution strategies

Previous research, summarized in the preceding section, shows that young children's strategies to solve elementary arithmetic word problems are

significantly influenced by the underlying semantic structure (see also Carpenter & Moser, 1982, 1984). However, one can question whether this factor is the only task characteristic responsible for the observed results. First, as we suggested elsewhere (De Corte & Verschaffel, in press), children's strategies may also be affected by other task characteristics that were not appropriately controlled in the previous studies, such as the relative size and the sequence of the given numbers in the problem text. For example, one could question the above-mentioned empirical evidence in favor of the impact of the semantic structure on children's verbal and mental solution strategies for addition problems, because the problems used in our study also differed with respect to the size of the difference between the two given numbers. Second, additional arguments to explore the impact of non-semantic factors originate from a well-known series of investigations in which response latencies were used to analyse children's solution procedures for numerical arithmetic problems (Groen & Parkman, 1972; Groen & Poll, 1973; Woods, Resnick & Groen, 1975; Suppes & Groen, 1967). Therefore, we did a new study, in which three additional task characteristics were involved, namely:

(1) The order of presentation of the two given numbers (larger or smaller first), e.g. "Pete had 3 apples; Ann gave him 9 more apples; how many apples does Pete have now ?" versus "Pete had 9 apples; Ann gave him 3 more apples; how many apples does Pete have now ?"

(2) The size of the difference between the two given numbers (large or small), e.g. "Pete had 3 apples; Ann gave him 9 more apples; how many apples does Pete have now ?" versus "Pete had 5 apples; Ann gave him 6 more apples; how many apples does Pete have now ?"

(3) The order of presentation of the two given sets in Change problems (regular versus inversed order), e.g. "Pete had 3 apples; Ann gave him 9 more apples; how many apples does Pete have now ?" versus "Ann gave Pete 9 more apples; he started with 3 apples; how many apples does Pete have now ?"

Some of these task variables have already been examined in previous research. For example, in studies by Nesher (1976) and by Rosenthal and Resnick (1974) elementary school children were given problems in which the sequence of actions was described in the chronological order and problems presenting the actions in a reversed order. Grunau (1978) tested whether word problems having a larger first addend were equal in difficulty to those in which the smaller addend was given first. However, most of these previous investigations studied the effects of these task characteristics only on problem difficulty. Surprisingly little research exists on their influence on children's solution strategies for word problems. Moreover, little or no effort has been made to relate the effects of these task characteristics to the problem's semantic structure.

5. Design, techniques and data sources

The present study consists of two parts: a paper-and-pencil test was administered to eighty five children; afterwards a smaller group of twenty pupils was individually interviewed.

The collective test contained twenty elementary addition and subtraction word problems representing four different types from the well-known classification scheme of Riley, Greeno and Heller (1983): Change 1 and Combine 1 (both involving addition) and Change 3 and Combine 2 (both involving subtraction). For each problem type, several variants were constructed by combining the three task characteristics mentioned above, namely the order of presentation of the two given numbers, the size of the difference between the two given numbers, and the order of presentation of the two given sets. However, one should note that the third characteristic - normal or inversed order of presentation of the two given sets - applies only for Change problems. Moreover, for Change 3 problems, this task characteristic coincides with the first one - the order of presentation of the two given numbers - as the start set necessarily contains the smallest given number (see Table 3 and 4).

Table 3 and 4

To neutralize the possibly disturbing influence of other task variables, three different versions of the test were made in which we altered (1) the order of presentation of the problems, (2) the names of the persons and the objects in the problems, and (3) the given numbers. In each version the following three number pairs were used for addition problems with a small and a large difference between the two given numbers respectively: (5,7), (7,8), (5,6) and (4,8), (5,9), (3,9). For the subtraction problems we used (13,9), (11,8), (12,9) and (11,3), (12,3), (13,4) respectively. The test was administered collectively in the beginning of the school year to four second-grade classes, with a total of eighty five children. They were not only asked to solve the problems, but also to write down for each problem a number sentence showing which arithmetic operation had been performed to find the solution.

In the middle of the next school year, a group of twenty first graders was individually interviewed using exactly the same set of word problems. Each problem was read aloud by the interviewer and the children were asked to perform the following tasks: to solve it, to explain the solution strategy, and to build a material representation of the problem using puppets and blocks.

Both the collective and the individual test provided systematic data on the relative difficulty of the distinct types of word problems, on the strategies children used to solve them and on the nature of their errors. In the present paper we focus on the influence of the task characteristics on children's solution strategies.

6. Hypotheses

In this section we present the main hypotheses concerning the influence of the distinct task characteristics on children's solution strategies for addition and subtraction problems respectively.

6.1 Hypotheses for addition problems

A first plausible hypothesis is that addition word problems in which the smaller addend is given first will be solved more frequently by S-strategies than problems in which the larger number is given first (Hypothesis A1).

Second, we hypothesize that the influence of the order of presentation of the two given numbers will interact with the size of the difference between both addends. More specifically, it is expected that problems starting with the smaller addend, will be solved more frequently with a S-strategy when there is a large difference between the addends (e.g. 3 and 9) than when both addends have almost the same size (e.g. 5 and 6); in the latter case the advantage of altering the order of both numbers and starting with the larger one, is almost nonexistant (Hypothesis A2a). For a similar reason, we expect that problems starting with the larger addend will be solved less frequently with a S-strategy when there is a large difference between the addends than when both addends have almost the same size (Hypothesis A2b).

Third, we hypothesize that the effect of the order of presentation of the two given numbers will also interact with the problem structure. In our previous work we have found that a Combine 1 problem in which the smaller number is given first, elicits more S- than F-strategies; for the corresponding Change 1 problem on the other hand, we observed more F- than S-strategies. However, as said before, we did not systematically control for the size of the difference between the two given numbers, nor did we take into account the order of presentation of the two given sets in the Change 1 problem. The design of the present study allows us to verify the earlier finding, and to compare the strategy choice for the regular and the inversed Change 1 problems. In this respect we expect - in line with the previous result - that there will be more S-strategies for Combine 1 than for regular Change 1 problems, when the smaller number is given first (Hypothesis A3a). The argument underlying this hypothesis is that the dynamic nature of the Change 1 addition problems will invite children to model the described chronological sequence of events in their solution strategy. Because Combine problems have no implied action, altering the order of the two given sets seems less problematic. For a similar reason - namely the tendency to solve Change problems using a strategy that parallels the sequence of events described in the problem - it is hypothesized that inversed Change 1 problems will elicit more strategies starting with the second given number - i.e. the start set -

than normal Change 1 problems, in which the start set is given first (Hypothesis A3b).

6.2 Hypotheses for subtraction problems

With respect to subtraction problems, we are especially interested in the influence of the distinct task characteristics on the choice of either a direct subtractive (DS) or an indirect additive (IA) solution strategy.

Based on arguments similar to those underlying our first prediction for addition problems, we hypothesize that subtraction problems starting with the larger given number will elicit more DS- and less IA-strategies than those in which the smaller number is given first (Hypothesis S1).

However, we assume that children's solution strategies on subtraction word problems will also be affected by another task characteristic, namely the size of the difference between the given numbers in the problem. In this respect, the earlier mentioned response latency studies with numerical problems suggest that children will try to apply that kind of strategy resulting in the shortest solution path (Groen & Poll, 1973; Woods et al., 1975). Based on these studies, we expect that subtraction word problems having a small difference between the two given numbers (e.g. 12-9=.) will elicit more IA-strategies than problems with a large difference (e.g. 12-3=.), for which a DS-strategy is more efficient (Hypothesis S2).

Finally, we hypothesize that the influence of the order of presentation of the given numbers and the size of their difference will interact with the semantic structure underlying the subtraction problem. More specifically, we expect that the implied joining action between the known start set and the unknown change set in Change 3 problems will elicit a large amount of IA-strategies, even when the order of presentation and/or the relative size of the numbers favour a DS-strategy. For Combine 2 problems, on the other hand, the choice of either a IA- or a DS-strategy will be influenced more obviously by the position and the size of the given numbers (Hypothesis S3).

7. Results

In analyzing the data with respect to the previous hypotheses we only took into account the correct solutions and those answers that could be considered as so-called technical errors, i.e. the child chooses the appropriate operation but fails in its execution. Children's strategies were derived from their written and verbal reports. Sometimes it was difficult or even impossible to infer what strategy had been applied to solve a particular problem on the basis of the available written or verbal protocol. It is obvious that such cases were also excluded from further analysis.

The findings on children's solution strategies for addition and subtraction problems are described consecutively. In this respect we mention that - as in all previous research on young children's elementary arithmetic word problem solving (see e.g. Briars & Larkin, 1984; Riley et al., 1983) - the percentages of children who answered a problem correctly or with a technical error were much higher for the addition (Change 1 and Combine 1) than for the subtraction problems (Change 3 and Combine 2) during both the collective and the individual test. All addition problems were solved correctly by at least 85 percent of the children; on the contrary, on the subtraction problems the percentages mostly varied between 40 and 65 percent (see Table 5 and 6). Consequently, the data on children's solution strategies for subtraction problems are based on a considerably smaller number of cases than those for addition.

Table 5 and 6

We finally point to the fact that the data with respect to our hypotheses were not submitted to a statistical analysis. The main reason is that our data on children's strategy choice for the distinct problem types are based on different numbers of children; moreover from problem to problem the children involved in the analysis were not the same.

7.1 Results on addition problems

Hypothesis A1. The first hypothesis states that addition problems in which the smaller number is given first, will be solved more frequently by S-strategies than problems in which the larger addend is given first. Table 7 gives the

percentages of F- and S-strategies on problems starting with the smaller and the larger given number during the collective and the individual testing situations.

Table 7

The results are in line with the hypothesis in both the collective and the individual tests. Children reported much more frequently that they had solved a problem with a S-strategy when it started with the smaller number than when the larger addend was given first. This finding suggests that children's solution strategies are indeed strongly influenced by the location of the smaller and the larger given number. Interestingly, the obvious tendency to begin the solution process with the first given number was much greater in the collective than in the individual test.

Hypothesis A2. It was expected that addition problems involving a large difference between the given numbers would evoke more S-strategies than those in which both addends have almost the same size, when the smaller number is given first (Hypothesis A2a); similarly, we hypothesized that problems in which the first given number is considerably larger than the second one, would elicit less S-strategies than problems in which the difference between both numbers is almost non-existant (Hypothesis A2b). Table 8 shows the percentages of F- and S-strategies for items with a small and a large difference between the two given numbers.

Table 8

Although the results are generally consistent with the predictions, they are not very convincing: for the collective test the observed difference in the amount of S-strategies between the small- and the large-difference condition is only 6 and 2 percent for problems starting with the smaller and the larger given number respectively. For the individual test the differences are somewhat larger, especially for problems starting with the larger given number (12 %). These data suggest that with respect to young problem solvers no considerable differences

in solution strategies occur depending on the size of the difference between the two given numbers.

Hypothesis A3. Finally, it was predicted that Combine 1 problems would elicit more S-strategies than normal Change 1 problems when the smaller number is given first (Hypothesis A3a), and also that inversed Change 1 problems would provoke more S-strategies both when the smaller and the larger number is given first as compared to normal Change 1 problems (Hypothesis A3b).

Table 9

The results in Table 9 are in line with both hypotheses. With respect to problems starting with the smaller given number, children seemed to find it easier to use the more efficient S-strategy in the context of Combine 1 and inversed Change 1 problems, than when the problem had a normal Change 1 structure, especially on the individual test; the same trend occurs in the collective test results, although less strong. With respect to the problems in which the larger number is given first, inversed Change 1 problems obviously elicited the highest percentage of S-strategies in the individual test situation; the difference is again in the predicted direction on the collective test, but very small.

7.2 Results for subtraction problems

The most remarkable finding for the subtraction problems was children's apparently very strong tendency to use IA-strategies, especially during the individual tests: on a total of 78 appropriate solution strategies, only four DS-strategies were observed. Consequently, the further discusssion is restricted to the data of the collective test.

Hypothesis S1. It was hypothesized that the choice of either a direct subtractive (DS) or an indirect additive (IA) strategy would be influenced by the relative size of the first given number: problems starting with the smaller

number would elicit more IA-strategies and less DS-strategies than problems in which the larger number is given first.

The results shown in Table 10, are in line with this prediction: we observed considerably more IA-strategies for problems starting with the smaller given number than for problems in which the larger number was given first. This finding supports the hypothesis that the order of presentation of the two given numbers has an influence on the kind of strategies children use to solve subtraction problems.

Table 10

Hypothesis S2. According to the second hypothesis, children would choose different strategies for problems with a small and a large difference between the two given numbers. More specifically, it was predicted that more DS- and less IA-strategies would be observed for problems with a small difference than for problems with a large difference.

Table 11

As Table 11 shows, there was no difference in the amount of DS- and IA-strategies for both types of problems. This result suggests that the relative size of the two given numbers has no significant influence on the strategies children use to solve subtraction problems. This conclusion complements the similar finding for addition problems (Hypothesis A2).

Hypothesis S3. Finally, it was expected that the effects of the order of presentation and the relative size of the numbers would interact with the semantic structure of the problem. More specifically, we assumed that the influence of these task characteristics will be greater for Combine 2 problems than for Change 3 problems.

Table 12

The results appear in Table 12 and show that Combine 2 problems starting with the smaller and the larger given number elicited indeed considerably different percentages of DS- and IA-strategies: while Combine 2 problems beginning with the smaller given number were solved much more frequently with IA- than with DS-strategies, the percentages of IA- and DS-strategies were much closer when the larger number was given first. For Change 3 problems starting with the smaller and the larger given number, the distribution of DS- and IA-strategies was almost alike: most children continued to apply IA-strategies even when the larger number was given first. These findings confirm the hypothesis that the influence of the order of presentation of the given numbers is not alike for all semantic problem types.

6. Discussion

Over the past few years a substantial body of research has yielded evidence that the semantic structure of simple addition and subtraction word problems seriously influence children's solution processes (Briars & Larkin, 1984; Carpenter & Moser, 1982, 1984; De Corte & Verschaffel, 1987; Nesher, 1982; Riley et al., 1983). The results of this study are certainly not in conflict with this well-documented finding but rather complementary. Indeed, our data show that with respect to young problem solvers considerable differences in solution strategies can occur within a given semantic problem type, depending on other task characteristics, i.e. the position of the two given numbers and the order of presentation of the sets in the problem text. Moreover, the findings reveal that the effects of these two additional task characteristics on children's solution strategies are not alike for all semantic problem types. These findings are not only helpful in explaining apparently conflicting results from different previous empirical studies involving the same types of word problems, but they also provide guidelines for improving and elaborating the available theoretical (computer) models of young children's skill in solving elementary arithmetic word problems.

Our data do not provide evidence supporting the impact on children's strategy choice of the third additional task variable involved in this study, namely the

size of the difference between the two given numbers. The importance of this task characteristic has originally been raised in the context of children solving numerical problems using verbal counting strategies. However, verbal counting is not the only way of solving elementary addition and subtraction problems; as we have seen, there are also material and mental solution strategies. Furthermore, the available research has not produced strong empirical evidence in favour of the influence of the size of the difference between the given numbers on children's strategy choice. Finally, it is plausible to assume that the impact of this task characteristic is less important with respect to word problems as compared to numerical problems.

Considerable differences were found between the collective and the individual tests. Several factors may contribute to explain this fact, such as the age of the children (first versus second graders), and the mathematics program used in each group. Probably a more important factor relates to the way in which we tried to identify children's solution strategies. In the individual interviews observational data together with thinking-aloud and retrospective data were used to unravel the child's solution strategy; however, in the collective test we could only rely on the child's written response to the instruction to write down his solution strategy. The latter approach is subjected to several problems. First, many young children have considerable difficulties in connecting their informal solution strategies to the formal-mathematical symbols and rules involved in number sentences. Consequently, a child may fail to write down an appropriate number sentence for a word problem he was able to solve (Carpenter, Hiebert & Moser, 1983; De Corte & Verschaffel, 1985; Lindvall & Ibarra, 1980). A second problem relates to the ambiguity of the sentence-writing task. Indeed, number sentences can fulfil two different functions with respect to word problems: they can be used either as a formal-mathematical representation of the semantic relations between the known and the unknown quantities in the word problem, or as the mathematical notation of the operation that has been performed with the two given numbers to find the solution of the problem. Sometimes, the same number sentence can fulfil both functions; in other cases, both aspects have to be expressed by different number sentences (De Corte & Verschaffel, 1985; Vergnaud, 1982). Although the children were instructed to write down the numerical sentence that shows how they solved the problem, some

may have preferred to state the one that represents the problem structure. This may explain why we found so many strategies beginning with the first given number in the collective as compared to the individual test.

It is clear that more theoretical and empirical work is required to clarify the relationship between the different task characteristics of word problems and children's solution strategies. First, more empirical research is needed in which the results of the present investigation are replicated with more tasks and larger samples of subjects. Second, the present study provides a relatively static view of children's solution processes. Further research is needed focusing at the effects of different task characteristics on children's strategy choice in different developmental stages. Third, one should investigate to what extent children are aware of the factors that determine their strategy choice and how this relates to their knowledge of mathematical principles such as the commutativity principle and the complementarity of addition and subtraction.

Note

(1) We point to the fact that in that longitudinal investigation a slightly different terminology was used, namely S(maller)- versus L(arger)-strategies, as in all eight problems the smaller number was always given first.

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Table 1. Examples of F- and S-strategies for addition problems (Problem : "Pete had 3 apples; Ann gave him 8 more apples; how many apples does Pete have now ?")

F-strategies	S-strategies
Material	
The child makes a set of 3 blocks; then makes a set of 8 blocks and adds this set to the first; finally the union is counted starting with one.	The child makes a set of 8 blocks; then makes a set of 3 blocks and adds this set to the first; finally the union is counted starting with one.
Verbal	
Counting-on-from-first: the child begins a counting sequence starting with the first given number and continuing the number of units represented by the larger number (3,... 4,5,6,7,8,9,10,11)	Counting-on-from-second: the child starts a counting sequence beginning with the second given number and continuing the number of units represented by the smaller number (8,... 9,10,11).
Mental	
The answer is derived from a recalled number fact beginning with the second given number (3+8=3+7+1= 10+1=11).	The answer is derived from a recalled number fact beginning with the second given number (3+8=8+2+1=10+ 1=11).

Table 2. Examples of DS- and IA-strategies for subtraction problems (Problem : "Joe has 3 balloons; Connie has 11 balloons; how many more balloons does Connie have than Joe ?")

Direct subtractive	Indirect additive
Material	
Separating from : a set of 11 objects is constructed; 3 objects are removed; the answer is the number of remaining objects.	Adding on : A set of 3 objects is constructed; objects are added to this set until there is a total of 11 objects; the answer is the number of elements added.
Verbal	
Counting-down-from-given : a backward counting sequence is initiated starting with 11; the sequence contains 3 counting words (11,...10,9,8); the last number in the counting sequence is given as the answer.	Counting-up-from-given : a forward counting sequence is initiated starting with 3 and continues until 11 is reached (3,...4,5,6,7,8, 9,10,11); the answer is the number of counting words in the sequence.
Mental	
The answer is derived from a subtraction fact recalled from memory (11-3=11-1-2=10-2=8).	The answer is derived from an addition fact recalled from memory (3+7 +1=11 thus 3+8=11).

Table 3. Overview of the addition problems used in the study

Structure	Sequence	First	Difference	Problem
Change 1	Normal	Larger	Large	Pete had 8 apples; Ann gave Pete 4 more apples; how many apples does Pete have now ?
Change 1	Normal	Larger	Small	Pete had 6 apples; Ann gave Pete 5 more apples; how many apples does Pete have now ?
Change 1	Normal	Smaller	Large	Pete had 4 apples; Ann gave Pete 8 more apples; how many apples does Pete have now ?
Change 1	Normal	Smaller	Small	Pete had 5 apples; Ann gave Pete 6 more apples; how many apples does Pete have now ?
Change 1	Inversed	Larger	Large	Ann gave Pete 8 more apples; Pete started with 4; how many apples does Pete have now ?
Change 1	Inversed	Larger	Small	Ann gave Pete 6 more apples; Pete started with 5; how many apples does Pete have now ?
Change 1	Inversed	Smaller	Large	Ann gave Pete 4 more apples; Pete started with 8; how many apples does Pete have now ?
Change 1	Inversed	Smaller	Small	Ann gave Pete 5 more apples; Pete started with 6; how many apples does Pete have now ?
Combine 1	-	Larger	Large	Pete has 8 apples; Ann has 4 apples; How many apples do Pete and Ann have altogether ?
Combine 1	-	Larger	Small	Pete has 6 apples; Ann has 5 apples; How many apples do Pete and Ann have altogether ?
Combine 1	-	Smaller	Large	Pete has 4 apples; Ann has 8 apples; How many apples do Pete and Ann have altogether ?
Combine 1	-	Smaller	Small	Pete has 5 apples; Ann has 6 apples; How many apples do Pete and Ann have altogether ?

Table 4. Overview of the subtraction problems used in the study

Structure	Sequence	First	Difference	Problem
Change 3	Normal	Smaller	Large	First Pete had 4 marbles; now Pete has 13 marbles; how many marbles did Pete win ?
Change 3	Normal	Smaller	Small	First Pete had 9 marbles; now Pete has 13 marbles; how many marbles did Pete win ?
Change 3	Inversed	Larger	Large	Now Pete has 13 marbles; first Pete had 9 marbles; how many marbles did Pete win ?
Change 3	Inversed	Larger	Small	Now Pete has 13 marbles; first Pete had 4 marbles; how many marbles did Pete win ?
Combine 2	-	Smaller	Large	Pete has 4 car toys; Pete and Ann have 13 toy cars together; how many toy cars does Ann have ?
Combine 2	-	Smaller	Small	Pete has 9 car toys; Pete and Ann have 13 toy cars together; how many toy cars does Ann have ?
Combine 2	-	Larger	Large	Pete and Ann have 13 toy cars; Pete has 4 toy cars; how many toy cars does Ann have ?
Combine 2	-	Larger	Small	Pete and Ann have 13 toy cars; Pete has 9 toy cars; how many toy cars does Ann have ?

Table 5. Overview of the level of difficulty of the addition problems used in the study

Structure	Sequence	First	Difference	Problem difficulty	
				Collective test	Individual test
Change 1	Normal	Larger	Large	95	100
Change 1	Normal	Larger	Small	95	95
Change 1	Normal	Smaller	Large	93	95
Change 1	Normal	Smaller	Small	95	95
Change 1	Inversed	Larger	Large	91	95
Change 1	Inversed	Larger	Small	95	95
Change 1	Inversed	Smaller	Large	99	100
Change 1	Inversed	Smaller	Small	93	85
Combine 1	-	Larger	Large	96	95
Combine 1	-	Larger	Small	93	95
Combine 1	-	Smaller	Large	98	95
Combine 1	-	Smaller	Small	99	100

Table 6. Overview of the level of difficulty of the subtraction problems used in the study

Structure	Sequence	First	Difference	Problem difficulty	
				Collective test	Individual test
Change 3	Normal	Smaller	Large	53	45
Change 3	Normal	Smaller	Small	54	65
Change 3	Inversed	Larger	Large	46	40
Change 3	Inversed	Larger	Small	53	75
Combine 2	-	Smaller	Large	53	55
Combine 2	-	Smaller	Small	58	50
Combine 2	-	Larger	Large	68	35
Combine 2	-	Larger	Small	69	40

Table 7. Percentage of F- and S-strategies on addition problems starting with the smaller and with the larger given number during the collective and the individual test

First given number	Collective test		Individual test	
	Strategies		Strategies	
	F	S	F	S
Smaller	85	15	43	57
Larger	96	4	78	22

Table 8. Percentage of F- and S-strategies on addition problems with a small and a large difference between the two given numbers during the collective and the individual tests

First given number	Difference between given numbers	Collective test Strategies		Individual test Strategies	
		F	S	F	S
Smaller	Large	82	18	40	60
	Small	88	12	45	55
Larger	Large	95	5	84	16
	Small	97	3	72	28

Table 9. Percentage of F- and S-strategies on normal Change 1, inversed Change 1 and Combine 1 problems during the collective and the individual tests

First given number	Structure	Collective test Strategies		Individual test Strategies	
		F	S	F	S
Smaller	Change 1 normal	91	9	52	48
	Change 1 inversed	77	23	39	61
	Combine 1	86	14	37	63
Larger	Change 1 normal	97	3	86	14
	Change 1 inversed	92	8	66	34
	Combine 1	99	1	83	17

Table 10. Percentages of DS- and IA-strategies on subtraction problems starting with the larger and with the smaller given number during the collective test

First given number	DS-strategies	IA-strategies
Larger	33	67
Smaller	17	83

Table 11. Percentages of DS- and IA-strategies on subtraction problems with a large and a small difference between the two given numbers during the collective test

Difference between given numbers	DS-strategies	IA-strategies
Large	26	74
Small	25	75

Table 12. Percentages of DS- and IA-strategies on subtraction problems with a Combine 2 and a Change 3 structure during the collective test

Structure	First given number	DS-strategies	IA-strategies
Change 3	Smaller	16	84
	Larger	22	78
Combine 2	Smaller	18	82
	Larger	43	57